Patent citation analysis can be used as a method of apportionment in estimating patent value. Academic studies have shown a positive relationship between the value of a patent and the number of times it is cited by future patents. In a litigation context, courts have admitted citation analysis for use in analyzing reasonable royalty damages, but have refused to admit incorrect implementations of the methodology. An analysis of various apportionment methods demonstrates that proper implementation is key to a reliable damages estimate in patent infringement cases. Reasonable royalty damages are meant to approximate the value of a license that would have resulted from a hypothetical negotiation between the patent owner and the alleged infringer. Determining a reasonable royalty is complicated in cases involving multicomponent products that contain several valuable features unrelated to the patents at issue. In recent decades, courts have increasingly focused on apportionment to ensure that patent holders are adequately compensated—but not overcompensated—for the patented invention’s “footprint.” Depending on the facts of the case and the available data, a variety of apportionment approaches may be available to the damages expert. Comparable license agreements are often used as the basis for the damages expert’s reasonable royalty rate. However, these license agreements frequently cover a portfolio of patents in addition to the patents at issue in the case. Thus, the damages expert is often challenged to parse out the portion of the licensed patent portfolio royalty rate that relates to the patents at issue. Patent citation analysis (also referred to as forward citation analysis) can be an effective method for discerning the relative value of a patent within a licensed portfolio. As part of the process to demonstrate that a new patent represents a novel advancement, the new patent references prior patents in its patent application. Referencing these existing patents generates a “forward citation” for the cited patent. Academic studies have shown a correlation between forward patent citations and patent value. Thus, apportioning the value of a patent portfolio to its individual patents relies on the notion that, holding all else equal, valuable patents are cited more often than less valuable patents. Court decisions in patent cases suggest that the admissibility of citation analysis depends on the methodology’s implementation. When correctly implemented, citation analysis can be a powerful tool to assist in calculating economic damages. This article discusses the theory and practice of citation analysis, and how it can be relevant to the work of damages experts and valuation analysts in a litigation context.

Academic Research Background
The use of citation analysis for valuation analyses and economic damages calculations in litigation is fundamentally based on the assumption that a patent with more forward citations is more valuable than a patent cited less frequently (all else being equal). As discussed below, empirical academic studies support this assumption by finding a positive relationship between the value of a patent and the frequency with which it is cited by subsequent patents.

1 The notion of a hypothetical negotiation is encapsulated in Georgia-Pacific Factor 15: “The amount that a licensor (such as the patentee) and a licensee (such as the infringer) would have agreed upon (at the time the infringement began) if both had been reasonably and voluntarily trying to reach an agreement.” Georgia-Pacific v. U.S. Plywood Corp., 318 F. Supp. 1116, 1120 (S.D.N.Y. 1970).
2 Given that many expert reports are filed under seal or partially redacted, it is difficult to know how often citation analysis is used relative to other methods. However, when searching federal dockets of patent cases for various methods of apportionment over the prior 20 years, the phrase “citation analysis” occurred within approximately 6 percent of document entries that clearly identified an apportionment method. This suggests that citation analysis is regularly used by experts in litigation, but is not the dominant method used for apportionment analysis. The appropriate method for a given case will depend on its facts, as well as the documents and data available to the expert.
3 Conceptually, this is analogous to how seminal academic works are often cited more frequently (when compared to less important works) through footnotes or bibliographic references by subsequent publications on a particular subject.
Patent citation frequency has been associated with “technological significance” and importance for decades. Early research found that “patents, which underlay technically important products, were more than twice as frequently cited.” The authors viewed this result as implying “that the location and analysis of groups of highly cited patents can provide a valid indicator of patent areas of technical importance.” Subsequent researchers similarly concluded that “highly cited patents are of greater technical importance than less frequently cited patents … and thereby supports the growing use of patent citation indicators in assessing the technological importance of a company’s patent portfolio.”

Building on these early research investigations into patent citation frequency, economists first reported more than three decades ago that “patent citations may be indicative of the value of innovations.” Economic research has found that the number of patent citations is correlated with corporate stock market valuations, pharmaceutical company financial performance, and agricultural yields, among other estimates of economic value. A published academic literature review of prior studies reported that “forward citation intensity is, in fact, correlated with economic value.”

Given the connection between patent value and citation frequency, valuation practitioners and researchers have used citation counts to inform various economic analyses. Among other examples, citation analysis can help with a company’s internal management of its own intellectual property (e.g., identifying underutilized patents that other companies may be interested in). The technique can also assist with cross-licensing negotiations (e.g., identifying key patents, informing a company’s relative bargaining position, and “to decide who should pay whom and how much”). Researchers have also proposed performing company valuations based on an analysis of its patents.

Citation analysis can also be relevant to analyzing mergers and acquisitions, including “targeting, due-diligence, compatibility and valuation.” Academic researchers have found that patents with higher citations are more likely to be acquired by other firms, and they have used patent citation data to measure innovation activity and understand the motivations behind mergers.

**Patent Citation Analysis In Litigation**

In addition to the above examples, citation analysis has been used in litigation to assist with estimating damages from alleged patent infringement. In these cases, the defendant is liable for “damages adequate to compensate for the infringement, but in no event less than a reasonable royalty for the use made of the invention by the infringer, together with interest and costs as fixed by the court.” The idea of apportionment of patent infringement damages has existed for over a century, but the concept received renewed focus by damages experts in the last decade, as courts emphasized that a reasonable royalty analysis requires an expert to tie damages to the claimed invention’s footprint. This issue is especially important for products...

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6 Ibid.
7 Michael B. Albert et al., “Direct Validation of Citation Counts as Indicators of Industrially Important Patents,” Research Policy 20, no. 3 (June 1991): 251–259, https://doi.org/10.1016/0048-7333(91)90055-U.
10 Adam B. Jaife and Gaëtan de Rassenfosse, “Patent Citation Data in Social Science Research: Overview and Best Practices,” Journal of the Association for Information Science and Technology 68, no. 6 (June 2017): 1360–1377, https://doi.org/10.1002/asi.23731.
12 Ibid.
13 Ibid.
18 Garrettson v. Clark, 111 U.S. 120, 4 S. Ct. 291 (1884).
with multiple elements, as one court summarized: “In any case involving multi-component products, patentees may not calculate damages based on sales of the entire product, as opposed to the smallest salable patent-practicing unit, without showing that the demand for the entire product is attributable to the patented feature.”20 When the smallest salable patent-practicing unit incorporates technologies beyond those with a close relation to the patents at issue, additional apportionment is typically required.21

Citation analysis has been applied successfully by damages experts in the apportionment and calculation of damages in the form a reasonable royalty. Several courts have ruled that a proper application of citation analysis is admissible under the Daubert standard, with one court recently noting that several courts “have stated that patent citation analysis can be a sufficiently reliable method. … It is used by experts in the field to determine patent value.”22 While specific misapplications of citation analysis have been excluded under the Daubert standard, the authors are not aware of any court rejecting the methodology outright. The various case examples and Daubert decisions described below can help practitioners understand which critiques of the methodology ultimately go to the weight of the expert’s opinion and which critiques speak to the admissibility of the method.

In Finjan v. Blue Coat Systems, the court granted the defendant’s motion to exclude the plaintiff’s expert’s analysis that utilized citation analysis.23 In this case, the plaintiff’s expert used the number of forward citations for each patent to estimate the value of each patent-in-suit relative to each other, as opposed to analyzing the value of the accused features as a portion of the accused product. The court criticized the expert for implementing citation analysis in a manner that did not take into account the patent’s age and self-citations.24 Thus, the court concluded that “although a qualitative analysis of asserted patents based upon forward citations may be probative of a reasonable royalty in some instances, the Court finds that [plaintiff’s expert’s] application of the analysis in this case must be rejected” because their “straightforward application of a forward citation analysis without taking into consideration these potential problems renders the method unreliable for failure to specifically tie the methodology to the facts of this case.”25

In Realtek Semiconductor Corp. v. LSI Corp, the defendant’s damages expert calculated a reasonable royalty that began with a benchmark license for a patent portfolio that did not include the patents-in-suit.26 The defendant’s expert used citation analysis to estimate the relative values of the benchmark patent portfolio and the plaintiff’s patent.

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22 Forward citation analysis can be a reliable method to decide the relative value of patents and to assist in determining a reasonable royalty); Evolved Wireless, LLC v. Apple Inc., Case No. 15-543-JFB-SRF (D. Del. Mar. 12, 2019) (“The forward citation method has generally been regarded as reliable.”); Better Mouse Co. v. Steelseries Aps, Case No. 2:14-cv-198-RSP (E.D. Tex. Jan. 4, 2016) (“Forward citation analysis can be both relevant and reliable.”).
24 Self-citations occur when inventors subsequently cite their own patents in later patents.
25 Finjan, Inc. v. Blue Coat Systems, Inc.
portfolio. The expert then assumed all patents within the plaintiff’s portfolio were equally valuable and attempted to apportion the value of plaintiff’s patent portfolio down to the patents-in-suit by splitting the value equally among the patents within the portfolio. The court excluded this portion of the defendant expert’s analysis because it did not properly apportion the value to the patents-in-suit, writing that although the expert’s patent citation “method may be an acceptable methodology to determine the relative value of patent portfolios ... the fact that she applied this methodology to determine the value of [plaintiff’s entire] portfolio but then did not use it in calculating the value of the two patents-in-suit results in a skewed and misleading analysis.”

In *Manufacturing Resources International v. Civiq Smartscapes*, the court allowed the defendant’s expert’s use of citation analysis, noting that “[f]orward citation analysis can be a reliable method to decide the relative value of patents and to assist in determining a reasonable royalty.” In this case, the defendant’s expert relied on a prior royalty license that included a patent application to which multiple patents (including the patents-in-suit) claimed priority, and then used citation analysis to apportion the relative value of the patents-in-suit versus all other technologically similar patents.

Another court also allowed the defendant’s expert’s use of citation analysis in *Better Mouse Company v. Steelseries*, noting that “forward citation analysis can be both relevant and reliable.” Similar to *Manufacturing Resources International*, the defendant’s expert relied on a license agreement that included multiple patents, and used citation analysis to apportion down to the relative value of the patents-in-suit. The court specifically noted that the defendant’s expert accounted for the patents’ age and technology when performing citation analysis, writing that the expert’s implementation was “sufficiently relevant and reliable.” The court emphasized that “to the extent Plaintiff claims that forward citation analysis is never relevant for patent valuation, the Court rejects that claim. No binding authority states that forward citation analysis is per se not relevant to the facts of any case. Instead, the Federal Circuit has said that ‘damages models are fact-dependent.’”

The above court rulings demonstrate that citation analysis can be admitted as part of expert testimony on apportionment and reasonable royalty damages when the methodology is applied correctly. However, citation analysis has also been excluded when it is incorrectly applied and does not fit the facts of the case. As summarized by one economist, “Patent Citation Analysis is an important tool for courts to determine reasonable royalty awards.” In the following section, we present different implementations of citation analysis to show why failure to account for a patent’s age and technology can result in damage awards that are materially incorrect when compared to an implementation that courts have regularly ruled is admissible.

**Example Application**

One method of estimating reasonable royalty damages involves reliance on comparable licenses. However, it is often the case in practice that a portfolio of patents is licensed in prior agreements and the patents-in-suit may represent only a subset of the licensed portfolio. In this context, the expert may need to apportion the royalty rate from the portfolio of patents to assign a royalty rate individually to the patents-in-suit. The *Manufacturing Resources International* and *Better Mouse Company* cases indicate that a properly implemented citation analysis may support an apportionment analysis by quantifying the relative value of patents, thereby assisting in determining a reasonable royalty.

As a hypothetical example, suppose an expert identifies a comparable prior license agreement with a royalty rate of 5 percent for a six-patent portfolio that includes the patents-in-suit alongside other patents. A damages expert who simply splits the royalty rate equally (approximately 16.7 percent of the royalty rate attributable to each patent in the portfolio) risks exclusion under Daubert, as illustrated by the *Realtek Semiconductor* case. Similarly, there is Daubert risk to apportioning value to each patent using raw forward citation count, as noted in the *Finjan* case, because it does not account for other factors, such as a patent’s age. The court’s concerns in these cases are not just theoretical, as older patents have more citations than younger patents (all else being equal) and citation rates can vary by class of technology. Below, we demonstrate that overly

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27 Based on the *Realtek* Opinion, it appears the expert’s analysis simply used a raw citation count, although it is not entirely clear whether it also controlled for other relevant factors (e.g., patent age) that were discussed in relation to the exclusion of the expert’s analysis in *Finjan*.

28 *Realtek Semiconductor Corp. v. LSI Corp.*


31 Ibid.

32 Ibid.

simplistic approaches can result in a material inaccuracy when compared to implementing citation analysis in a manner that controls for a patent’s age and technology.

One method of accounting for patent age and technology involves first identifying a cohort of patents that are similar in age and technology. Next, forward citation count is normalized to the cohort’s median, which is an approach that is consistent with prior academic researchers and practitioners. For example, if a patent’s cohort has a median of 25 citations, then a patent with a citation count of 50 is rescaled to two, meaning it is cited twice as often as its peer group. After normalizing citation count for each patent, the portfolio royalty rate from the comparable license is then apportioned to individual patents (including the patents-in-suit) to estimate the relative royalty rate attributable to each patent.

Table 1 summarizes the relative value of each patent in a hypothetical six-patent portfolio based on four different implementations. Scenario 1 attributes equal value to each patent in the portfolio, akin to the Realtek Semiconductor case described above. Scenario 2 apportions value to each patent using a simple raw citation count, which is subject to the court’s criticism in the Finjan case described above. Scenario 3 normalizes citations to account for a patent’s age and technology in a manner consistent with the expert analysis that was admitted in the Manufacturing Resources International and Better Mouse Company cases described above. Scenario 4 adjusts the prior scenario for self-citation counts, which was an issue raised in at least one prior case, although the authors are not aware of a court ruling that an analysis is or is not admissible based on the issue of self-citations alone.

Table 1: Percentage Valuation Attributable to Each Patent in a Hypothetical Licensed Portfolio

<table>
<thead>
<tr>
<th>Patent</th>
<th>Month Issued</th>
<th>Scenario 1</th>
<th>Scenario 2</th>
<th>Scenario 3</th>
<th>Scenario 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>November 2000</td>
<td>16.7%</td>
<td>41.3%</td>
<td>21.3%</td>
<td>20.4%</td>
</tr>
<tr>
<td>B</td>
<td>October 2004</td>
<td>16.7%</td>
<td>13.9%</td>
<td>12.9%</td>
<td>12.3%</td>
</tr>
<tr>
<td>C</td>
<td>November 2005</td>
<td>16.7%</td>
<td>16.7%</td>
<td>21.1%</td>
<td>23.4%</td>
</tr>
<tr>
<td>D</td>
<td>June 2006</td>
<td>16.7%</td>
<td>22.5%</td>
<td>31.3%</td>
<td>31.3%</td>
</tr>
<tr>
<td>E</td>
<td>August 2008</td>
<td>16.7%</td>
<td>2.8%</td>
<td>5.6%</td>
<td>5.5%</td>
</tr>
<tr>
<td>F</td>
<td>January 2010</td>
<td>16.7%</td>
<td>2.8%</td>
<td>7.8%</td>
<td>7.1%</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>100.0%</td>
<td>100.0%</td>
<td>100.0%</td>
<td>100.0%</td>
</tr>
</tbody>
</table>

As demonstrated by Table 1, the percentage valuation attributable to each patent in the hypothetical portfolio differs substantially when assuming an equal split (Scenario 1) or using a raw citation count (Scenario 2), compared to scenarios that account for the patent’s age and technology (Scenarios 3 and 4). For this hypothetical patent portfolio, equally splitting the value of the license among the patents often bears no similarity to an approach that considers forward citation count to be a proxy for value (all else being equal). When using citation analysis but failing to account for the age of a patent, the oldest patent in the portfolio predictably has the highest share of forward citations, meaning it would have the highest implied royalty rate and incorrectly would yield the largest amount of damages. However, after controlling for patents’ age and technology, the issue of including self-citations matters less in this example.

35 Numbers shown may not total 100 percent in each instance due to rounding.
Table 2: Percentage Under/Overstatement in Damages Relative to Scenario 4 (accounting for patent’s age, technology, and self-citations)

<table>
<thead>
<tr>
<th>Patent</th>
<th>Scenario 1</th>
<th>Scenario 2</th>
<th>Scenario 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>-18.2%</td>
<td>102.8%</td>
<td>4.6%</td>
</tr>
<tr>
<td>B</td>
<td>35.8%</td>
<td>13.1%</td>
<td>4.9%</td>
</tr>
<tr>
<td>C</td>
<td>-28.8%</td>
<td>-28.7%</td>
<td>-9.8%</td>
</tr>
<tr>
<td>D</td>
<td>-46.8%</td>
<td>-28.2%</td>
<td>-0.0%</td>
</tr>
<tr>
<td>E</td>
<td>201.1%</td>
<td>-49.2%</td>
<td>1.0%</td>
</tr>
<tr>
<td>F</td>
<td>134.7%</td>
<td>-60.4%</td>
<td>10.2%</td>
</tr>
<tr>
<td>Average of absolute value of misstatement</td>
<td>77.6%</td>
<td>47.1%</td>
<td>5.1%</td>
</tr>
</tbody>
</table>

Table 2 shows the percentage by which damages would be misstated under Scenarios 1, 2, and 3 relative to Scenario 4. If only one patent is at issue in this hypothetical portfolio, Table 2 shows that assuming an equal split of value could understate damages by as much as 46.8 percent or overstate damages by as much as 201.1 percent relative to a citation analysis approach that accounts for a patent’s age, technology, and self-citations. Similarly, using a raw citation count and failing to account for a patent’s age, technology, and self-citations could understate damages by as much as 60.4 percent or overstate damages by as much as 102.8 percent. Although the average misstatement (in absolute value) declines from 77.6 percent to 47.1 percent when using a raw citation count instead of an equal value split, the misstatement is still material. However, the issue of self-citations appears to be less of a concern, with an average misstatement (in absolute value) of only 5.1 percent. This observation is corroborated by a recent study noting that “using forward citation counts can be sensitive to the issue of self-citations in some (but not all) cases.”

**Conclusion**

Academic studies have reported a positive relationship between a patent’s value and its forward citation count. As a result, citation analysis has been used to inform a variety of economic analyses, including an analysis of damages in patent infringement litigation.

Understandably, courts have been critical of the approach when it is applied incorrectly. But courts have ruled that the approach is admissible when correctly applied in a manner that fits the facts of the case. In this article, we demonstrate the impact of an improper implementation compared to an implementation that has been ruled admissible by the courts. Our analysis suggests that the courts in the referenced cases were justified in excluding citation analysis when it was improperly applied. However, when done properly, citation analysis can be a useful tool for calculating economic damages in patent infringement cases and a growing body of court decisions have rejected Daubert challenges when the analysis was implemented properly.

Daniel Werner, PhD, CPA, is an associate director at NERA Economic Consulting, where he provides economic, financial, and statistical analysis to support complex litigation. Dr. Werner regularly applies his expertise to matters involving economic damages and valuation issues, among other areas. He has also provided expert witness testimony in cases involving issues in intellectual property, false advertising, breach of contract, lost profits, lost wages, and fraud. Dr. Werner has applied his analyses to a wide range of industries, including consumer retail goods, grocery products, technology (hardware and software), and real estate. Email: daniel.werner@nera.com.

Huy Dang is a senior analyst in the Intellectual Property Practice at NERA Economic Consulting. He has worked on patent infringement, trade secrets misappropriation, and false advertising in a variety of industries, including automotive, software, drone, encryption, healthcare, and telecommunications. Email: huy.dang@nera.com.

36 Werner and Dang, “Patent Valuation Using Citations: A Review and Sensitivity Analysis” (see n. 34).