Introduction

Collateralized debt obligations (CDOs) and other structured financial products containing subprime mortgages have been a focal point of the credit crisis, giving rise to a growing amount of investigative journalism as well as credit crisis litigation. It is widely agreed that the leading edge of the credit crisis was the meltdown of the US subprime mortgage market that began in early 2007. Many of these mortgages were structured into asset-backed securities (ABS) that were then further structured into CDOs. Through these processes, exposures were propagated throughout the financial system, ultimately resulting in widespread losses. Concerns about the depth of these losses led to uncertainty about counterparty risk, causing the credit markets to freeze in August 2007. These fears were magnified in the financial market panic of September 2008 that followed the bankruptcy of Lehman Brothers. The government response had been massive, starting with bailouts and investigations by the Securities and Exchange Commission (SEC) and DOJ of potential wrongdoing, alongside an overhaul of financial regulation.

Given their prominent role, it is clear that disputes will continue to revolve around CDOs and other subprime-backed structured products for some time. Through the end of March 2010, the credit crisis had yielded at least 395 securities filings (excluding arbitrations). Of these, at least 41 are CDO-related, which includes suits by investors in CDOs; many others are suits by investors in various of their building blocks, such as ABS and credit default swaps (CDS).
Though many market participants were conversant with these structures, it will usually be the case that a lay audience does not have such familiarity. The goal of this paper is to go behind the current headlines to describe in plain English the fundamental analytics of the ABS-backed CDOs and synthetic CDOs that were instrumental in the financial crises. We will also discuss the principles of their valuation, including the important issue of correlation. While no short paper can cover the full breadth and detail of the subprime mortgage market, structured finance, and credit derivatives, the fundamentals presented here should help those who desire to increase their understanding of these topics.

Subprime, Securitization, and the Financial Crisis

“[G]iven the fundamental factors in place that should support the demand for housing, we believe the effect of the troubles in the subprime sector on the broader housing market will likely be limited, and we do not expect significant spillovers from the subprime market to the rest of the economy or to the financial system.”

-Federal Reserve Chairman Ben Bernanke, May 17, 2007

“[T]he economic outlook has been importantly affected by recent developments in financial markets, which have come under significant pressure in the past few months. The financial turmoil was triggered by investor concerns about the credit quality of mortgages, especially subprime mortgages with adjustable interest rates.”

-Federal Reserve Chairman Ben Bernanke, November 8, 2007

“The downturn in the housing market has been a key factor underlying both the strained condition of financial markets and the slowdown of the broader economy…. Despite the efforts of the Federal Reserve, the Treasury, and other agencies, global financial markets remain under extraordinary stress.”

-Federal Reserve Chairman Ben Bernanke, September 23, 2008

At the heart of the financial crisis are bank writedowns on CDOs, mortgage-backed securities (MBS), and ABS with mortgage collateral (called home equity ABS for reasons outlined in the next section). As the housing market declined and both subprime and prime mortgage delinquencies and defaults rose, these securities declined in value and became highly illiquid. Illiquidity and concern about the true value of CDOs and other structured products were the driving force behind TARP. The International Monetary Fund recently estimated that writedowns by US banks will total $885 billion between 2007 and 2010. Residential mortgage loans and securities account for over 40% of the estimated writedowns. Here, we briefly review how losses in the subprime mortgage market affected CDO values. As seen in Figure 1, delinquencies and foreclosures on subprime mortgages rose somewhat in 2006 and much more dramatically in 2007.
Losses, and fear of future losses, on subprime mortgage loans led to losses on home equity ABS containing those loans. The ABX indices, shown in Figure 2 (and described in detail below) track the value of a set of benchmark subprime-backed securities. The lower-rated BBB- index represents securities that are more sensitive to mortgage loan losses. While this index first began falling in response to problems in the subprime market, even the safest subprime-backed securities, tracked by the AAA index, began falling substantially in value as 2007 progressed.
In turn, losses, and fear of future losses on home equity ABS led to losses on CDOs containing those subprime ABS. By 2007 more than half of CDOs outstanding were what are called "structured finance CDOs"—the term for CDOs that contain structured finance securities including ABS, non-Agency MBS, and tranches of other CDOs. As we will discuss later in the paper, subprime-backed securities accounted for most of the collateral backing these CDOs.

The role of subprime in this turmoil is explained in large part by two factors. The first is that the design of subprime mortgages—many of which were originated with high loan-to-value (LTV) and debt-to-income (DTI) ratios—made them very sensitive to declines in housing prices (much more so than traditional mortgages). Defaults on large numbers of subprime loans followed slowdowns in the rate of change of housing prices. The second factor is that the risks underlying subprime mortgages were, through securitization and derivatives trading, distributed throughout the financial system. This reallocation and distribution of risks was believed to dissipate systemic risks. However, the large volume of home equity ABS and CDOs, and their purchase by a wide range of financial firms, caused the deterioration in subprime mortgages to affect balance sheets across the financial sector.

In what follows, we will describe the vehicles through which subprime mortgage risk was securitized and traded. These vehicles are ABS, CDS, and CDOs, which we discuss in turn. In doing so, we will explain the economics of these vehicles, which will help address important topics such as how to determine their worth and what investors were actually buying and selling.
Understanding Home Equity Asset-Backed Securities

Background: The securitization of mortgage loans

Securitization is the process of pooling together assets that are not readily tradable, such as mortgages, corporate loans, or credit card loans, and issuing securities that entitle investors to payments based on cash flows that come from the pool.

Prior to the advent of securitization in the 1970s, depository institutions (commercial banks and thrifts) were the predominant originators of residential mortgages. These institutions funded mortgage originations with deposits or by issuing bonds. By the early 1990s, a different kind of financial firm—mortgage bankers—grew to be the predominant originators of residential mortgages. These institutions would fund their lending with short-term lines of credit and repay the loans either by selling the whole loans to a housing agency (e.g., Fannie Mae), or by selling the mortgages into the secondary market, i.e., securitization. Today, over half of the $15 trillion in residential US mortgage debt is securitized. Under this originate-to-distribute business model, the bank, savings and loan association, or mortgage bank originating a mortgage then sells the mortgage to a trust. The trust enlists an underwriter and issues bonds (MBS or ABS) backed by the future cash flows of these mortgages. The trust also enlists a rating agency to place a rating on the MBS and ABS that it issues. The bank (or other originator) typically still services the mortgage (although another firm may do so), collecting payments and forwarding them to the trust, as well as dealing with any delinquencies or defaults. The cash flows from the mortgages are passed through to the purchasers of the bonds, which may include pension funds, insurance companies, mutual funds, hedge funds, or CDOs.

Housing agencies Fannie Mae, Freddie Mac, and Ginnie Mae have historically been responsible for most of the MBS issuance. Agency MBS are thought to be very safe in terms of credit risk or default risk; the agencies guarantee full and timely payment, and those guarantees are perceived to be backed by the US government. (In fact, only Ginnie Mae MBS are explicitly backed by the full faith and credit of the US government.) The agencies, though, are restricted as to what mortgage loans they can purchase and securitize. These loans are called “conforming” and meet certain criteria correlated with low historical risk of default.

The many non-conforming mortgage loans are securitized by other private institutions, like banks and mortgage lending companies; these include jumbo, Alt-A, and subprime. Jumbos are securitized pools of high credit quality mortgages whose loan sizes exceed the conforming limit. The mortgages that underlie Alt-A and home equity ABS, on the other hand, usually meet the conforming loan size limit. However, Alt-A mortgages are usually missing documentation, have minor credit problems, or both, while subprime mortgages generally suffer from substantial credit deficiencies.

Subprime mortgages are made to borrowers with a high DTI ratio, an impaired or minimal credit history, or other characteristics correlated with a higher probability of default. Because these borrowers are inherently riskier, subprime mortgages are originated at a premium above the prime mortgage rate offered to individuals with better credit attributes.
The classification of subprime loans and ABS has evolved over time. Residential ABS (as opposed to consumer ABS, such as securitized pools of auto loans or credit card receivables) are distinguished by the purpose of the loan and by the credit profile of the borrower. Residential ABS were once securitized by low balance, second lien mortgages to borrowers with prime credit. The sector became known as home equity loan ABS, or simply home equity ABS. By the late 1990s, the trend in the residential ABS market was toward first lien mortgages to subprime borrowers. Eventually, the sector of home equity ABS grew to encompass home equity lines of credit (HELOCs), high LTV mortgages, home improvement loans, and non/re-performing pools. By 2004, most home equity ABS transactions were backed by first lien mortgages to subprime borrowers. Because the terminology is standard in the industry, we refer to all ABS whose collateral consists of mortgages as home equity ABS.

The structure and economics of MBS and home equity ABS
The economics of MBS and ABS are defined by two types of risk. Default risk is the risk that the underlying mortgages will default and the bond investors will not receive their due interest and principal. Interest rate risk, comprised of prepayment and extension risk, is the second key risk type. MBS and home equity ABS can be characterized by how their structure protects certain investors from, or compensates other investors for bearing default and interest rate risks. Differences in structuring will affect how the default and prepayment risk of the mortgage pool is allocated among investors, and may mitigate the total risk. Investors who are allocated more of those risks naturally earn higher yields.

Agency MBS (and other MBS composed of conventional-type mortgages) are secured by fixed rate loans, and their interest and principal payments are guaranteed. For this reason, interest rate risk is central to the economics of these MBS. Prepayment risk is the risk that the mortgages will repay more quickly than anticipated. Fixed-rate mortgage borrowers tend to refinance in low interest rate environments, and when they refinance, the original mortgage is paid off completely. When rates fall, the investor in a bond backed by fixed-rate mortgages is repaid sooner than expected; but he would have preferred not to be, as he can now only reinvest that money at the prevailing lower interest rates instead of the higher interest rate he was receiving from the bond. Extension risk is the converse. This is when borrowers of fixed-rate mortgages—primarily as a result of rising rates—repay their obligations more slowly than anticipated. This extension can cause an investor to earn a lower than market rate of return for an extended period or take mark-to-market writedowns on the position.

Home equity ABS are backed by loans to riskier borrowers. For this reason, their economics centers around default risk. Because subprime collateral has higher default risk, home equity ABS are designed with a number of mechanisms called “credit enhancements” that attempt to mitigate and reallocate the default risk. Common credit enhancements are credit tranching, over-collateralization, excess spread, and monoline insurance. As credit tranching is the most common form of credit enhancement, and is fundamental to understanding CDOs as well as their meltdown, we discuss it in detail below.
While prepayment risk affects home equity ABS, it is more complicated than for the case of Agency MBS and other securities composed of conventional fixed-rate mortgages. Typically, subprime mortgages have a short fixed-rate period, after which the loan resets into a floating-rate mortgage. Subprime borrowers commonly refinance at the reset date. Investors in home equity ABS face the risk that borrowers do not refinance and repay principal at the reset date. The failure to refinance extends bond investors’ exposure to the default risk of the subprime borrowers. Moreover, their probability of default may rise after the reset date with the rise in their monthly payment obligations.

**Understanding tranching**
Credit tranching is a senior-subordinate credit structure, set up to provide certain investors in a particular structure with protection against defaults. Starting with a pool of mortgages, one creates two or more classes of bonds (the tranches), where the senior class has payment priority over the other, junior (subordinated) class. In a typical transaction, investors in the junior debt class may not receive any principal or interest until the senior classes have been paid in full. This priority of payments is often referred to as the “cash flow waterfall” and is illustrated in Figure 3. For a typical MBS deal, both the number of tranches and their subordination levels vary depending on the sector and the collateral.

Figure 3 shows a stylized example of an ABS deal with $100 million of collateral (which would be a pool of subprime mortgages, car loans, or other receivables) with four tranches. The senior piece has 20% subordination, meaning that it has rights to the first 80% of cash flows due from the underlying collateral. Assuming subordination is the only credit enhancement in the structure, the senior piece will not be affected unless more than 20% of scheduled principal is not received. The two “mezzanine” tranches have 12.5% and 10% subordination, and the “first-loss” piece (also called the equity tranche or residual) has no subordination at all. Because risks increase as we go down in the subordination ladder, bond coupons increase accordingly. As would be expected, more junior tranches receive lower credit ratings.

**Figure 3. ABS cash flow waterfall and credit tranching**

Payments from Collateral

<table>
<thead>
<tr>
<th>Collateral</th>
<th>$100 Mortgages</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Class A Principal</td>
<td>$80.00</td>
</tr>
<tr>
<td>Class B Principal</td>
<td>$7.50</td>
</tr>
<tr>
<td>Class C Principal</td>
<td>$2.50</td>
</tr>
<tr>
<td>Equity Tranche</td>
<td>$10.00</td>
</tr>
</tbody>
</table>

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By cushioning senior tranches from losses, credit tranching can create AAA-rated bonds out of a pool of risky collateral, including subprime mortgages. Suppose the collateral underlying the senior and junior tranches in the example above consists of subprime mortgages. Further, suppose that the loss on that collateral due to borrowers’ not making their mortgage payments turned out to be 15% of the $100 million of collateral, i.e., $15 million. Because the equity tranche and junior tranches are subordinated, they would receive no further payments. In addition, only $5 million would be left to pay to the Class B bondholders, as compared to $7.5 million in scheduled principal payments. In contrast, the senior tranche would have no loss, and the senior bondholders would receive their entire principal of $80 million. Losses could go as high as $20 million before the senior tranche would be exposed to any loss. Before conferring a AAA rating on the senior tranche, the rating agencies would estimate expected default losses, and they would consider even higher losses in stress scenarios. If, in the rating agency’s opinion, the senior tranche had only a small probability of experiencing credit losses, it could receive a AAA rating.

Although investors purchased MBS and ABS bonds directly, many of these securities ended up as collateral themselves inside CDOs. The structure of a CDO shares much in common with that of MBS and ABS. Valuing CDOs depends on understanding the risk in a portfolio of MBS and ABS.

Development and Structure of the Collateralized Debt Obligation Market Leading up to the Subprime Meltdown

CDO basics
CDOs have been around for two decades, but their growth as an asset class occurred primarily after 2000. As can be seen in Figure 4, issuance increased rapidly through the second quarter of 2007, followed by an even more rapid decline from an annual rate greater than $600 billion to a current annual rate of less than $5 billion. This inverted V-shaped pattern resembles a similar pattern in the issuance of ABS containing subprime mortgages (home equity ABS) and structured finance CDOs, as seen in Figure 5.

It is perhaps easiest to think of a CDO as a small, privately held finance firm with a finite life span. In fact, the CDO, which is structured as an independent legal entity, owns assets such as MBS, which generate cash flows to the CDO. These MBS cash flows are in turn derived from interest and principal payments made by the borrowers whose mortgages are owned by MBS structures. The CDO finances itself by issuing a number of debt classes and a thin slice of equity. A CDO entity’s debt is issued within a capital structure that includes a strict hierarchy of subordination. The cash flows from the underlying assets (i.e., the cash flows coming into the CDO structure) are distributed (i.e., sent out of the CDO structure) according to a “waterfall” that is described in the CDO’s offering memorandum. Interest and principal payments from the underlying MBS assets are paid to the various classes of debt and equity based on capital structure priority (or seniority). Continuing the analogy of the CDO as a firm, the tranches can be thought of like the various seniorities of a firm’s liabilities: senior and subordinated debt, preferred stock, and common stock.
Figure 4. Global CDO issuance ($ billions, quarterly)

Source: Thomson Reuters, SIFMA

Figure 5. Home equity ABS and Structured Finance (SF) CDO issuance ($ billions, annual)

Source: Thomson Reuters, SIFMA
CDO structure: Cash flow waterfalls and coverage tests
The ratings and safety of the various classes reflect their seniority as claimants on the cash flows from the underlying collateral. Junior classes will not get paid before the more senior tranches are paid.

The creditworthiness of the investors’ principal can receive additional protection from regularly scheduled coverage tests. These tests include a variety of over-collateralization and interest coverage ratios. When the tests are passed for a class, the tranche holders receive their scheduled interest payments as well as any principal repayments after the more senior classes (if any) have been paid down. If a test is failed, then the most senior class may be able to liquidate the entire capital structure and some junior classes may receive only partial payment or no payment at all.

The simplified structure of the typical CDO backed by ABS is shown in Figure 6, below. There, the CDO is divided into five tranches with coupons that increase as the seniority declines: a senior tranche that typically has a AAA rating, a junior tranche rated AA, a mezzanine A-rated tranche, a subordinated B-rated tranche, and an unrated equity tranche that is often retained by the issuer. The CDO has features similar to ABS. One of the differences, however, is that a CDO can have a wide variety of assets for collateral, including not only MBS and ABS, but also commercial MBS, real estate investment trust debt, corporate loans, and debt from a variety of different industries and geographies, and even tranches from other CDOs.

We will focus on CDOs with home equity ABS collateral—the type of most importance to the financial meltdown.\(^\text{17}\) Such collateral can be diversified by type of subprime mortgage (e.g., equity loan versus primary mortgage) and geography (e.g., Southwest versus Northeast).\(^\text{18}\) Alternatively, as is now well known, a number of CDOs were issued that were made up of only the A-rated and BBB-rated tranches of home equity ABS—so-called mezzanine CDOs. Even though the CDO was backed by higher yielding, lower credit assets, the tranching mechanism described above made it possible for the senior piece to achieve a AAA rating, though in a smaller ratio than non-mezzanine, ABS-backed CDOs. When the demand for pools of BBB-rated tranches of home equity ABS proved to be less than the demand for ABS CDOs, synthetic CDOs (described below) were created that referenced the BBB-rated ABS tranches but did not actually own them.
Economics of subprime mortgages and impact on CDOs

From 2000 to early 2007, when CDO issuance was rising rapidly, the subprime mortgage market issued a number of types of mortgages that, according to Gorton (2008), were designed to encourage early repayment and refinancing by the homeowner. One of the most common subprime mortgage products was given the label 2-28 because it offered a low fixed rate (called a “teaser rate”) for two years and then converted (reset) to a high variable-rate mortgage for the remaining 28 years. (Mortgage products that convert from fixed rate to floating rate are often called “hybrids.”) Homeowners who could afford the teaser rate, however, might have found themselves unable to afford the subsequent rate two years later. The jump from the teaser rate to the floating rate is greater with 2-28 mortgages as compared to other hybrid products. This reset shock meant that borrowers’ monthly payments could jump by 15% to 35% or more when the teaser rate expired, even if market interest rates were unchanged over the period from loan origination to reset. The interest rate jump after two years created a strong incentive, if not the need, for the homeowner to refinance into another 2-28 or similar mortgage product after two years.

The certainty of the rate jump made subprime mortgage delinquency and default rates particularly sensitive to housing prices—much more so than conforming mortgages. If housing prices were to fall, subprime borrowers would find themselves unable to refinance when their teaser rates expired. Because a significant number of subprime mortgages had LTV ratios of up to 100% (versus 80% or less for conforming mortgages), a decline in housing prices could render subprime borrowers “under water” and unable to refinance. The incentive to refinance after only two years gave subprime borrowers little flexibility to withstand even a short-lived drop in housing prices.

The expectation of refinancing after two years led to the belief that prepayment rates on the underlying CDO collateral would be relatively high, thereby causing relatively rapid principal pay-downs on the senior tranches. Indeed, the average life of the higher tranches of CDOs was expected to be less than five years. The principal pay-down feature, if it was triggered, would then lead to a virtuous cycle of increasing creditworthiness for the lower tranches.
This process is illustrated in Figure 7, which is based on the CDO structure from Figure 6, showing two periods in the life of a CDO—period 1 at issuance and period 2 some time later when 25% of the principal has been paid down based on mortgage refinancings and amortizing principal payments.

Figure 7. Waterfall diagram with attachment points

The columns of percentages alongside the tranche breakpoints in Figure 7 show what are called attachment points. This concept shows what percentage of losses can be sustained before a tranche becomes impaired; that is, when there is no longer enough collateral to repay the entire principal due to that tranche. As a CDO amortizes, in the absence of defaults, attachment points for each non-equity tranche will increase. In other words, each tranche’s effective subordination for remaining payments will increase as the bond amortizes. The equity tranche generally attaches at zero because any initial defaults reduce cash flows to equity. In the Figure 7 structure, the BBB tranche attaches at 3.0% at issuance (Period 1), the A tranche attaches at 7.5% and the AA tranche attaches at 12.5%. But with principal pay-downs to the senior class, and no defaults, the attachment points will all increase. In Figure 7, the assumed amount of the pay-downs is such that the attachment points all increase by a third. This, in turn, increases the value of the junior tranches because it means that a higher default rate on the underlying mortgages will be required before the principal of each tranche becomes impaired.

It should be recognized, however, that attachment points could—and eventually did—decrease for each tranche. If homeowners were to default rather than refinance, the attachment points would be at lower percentages, lowering the effective subordination for remaining payments; as a result, the creditworthiness of all the tranches in the structure would decline.
For this reason, some commentators have pointed out the importance of increasing housing values in supporting the CDO market through the years of rapid growth. Although future research may well show that there were other causal factors at work in the collapse of the CDO market following the housing market downturn, the importance of substantial, widespread (hence unanticipated) declines in housing prices as a precipitator will likely remain a key finding.

**Diversification of CDO collateral**

CDOs are designed to facilitate diversification because, in part, a diversified portfolio of debt assets will have less risk than its constituent assets and will further reduce risk for the higher tranches in the CDO structure. Given that a typical CDO contains 100 or more component securities, there is opportunity for substantial diversification and risk reduction. The extent to which CDO collateral was diversified, however, varied from one CDO to another. Moreover, the amount of diversification, and, therefore, the extent of risk reduction achieved, depends not on the number of different component securities, but on the correlation of default among those securities (which of course, can only be estimated). Specifically, if there is a high correlation of default risk among the collateral assets then risk will not be reduced as much as it would be if the correlation were low. The concept of correlation and its role in valuation of CDOs will be explored in more depth in later sections.

By 2006, the typical high-grade ABS CDO held 50% of its collateral in home equity ABS and the typical mezzanine CDO held 77% of its collateral in home equity ABS and 89% in residential MBS, as seen in Figure 8.

### Figure 8. Typical collateral composition of ABS CDOs

<table>
<thead>
<tr>
<th>Collateral</th>
<th>High-Grade ABS CDO</th>
<th>Mezzanine ABS CDO</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subprime RMBS</td>
<td>50%</td>
<td>77%</td>
</tr>
<tr>
<td>Other RMBS</td>
<td>25%</td>
<td>12%</td>
</tr>
<tr>
<td>CDO</td>
<td>19%</td>
<td>6%</td>
</tr>
<tr>
<td>Other</td>
<td>6%</td>
<td>5%</td>
</tr>
<tr>
<td>Total</td>
<td>100%</td>
<td>100%</td>
</tr>
</tbody>
</table>


Many CDOs contain tranches of other CDOs as part of their portfolios. Depending on what portion of the portfolio consists of other CDOs, these deals are sometimes referred to as CDO\(^2\) ("CDO-squared"). As of 2007, it was estimated that, on average, 6% of the collateral of Mezzanine SF-CDOs (i.e., CDOs comprising of structured finance products, like investment-grade MBS and other ABS) consisted of tranches of other CDOs. High-grade SF-CDOs held 19% of collateral in other CDOs in the same year. Including tranches of other CDOs was considered a way to further diversify exposure to the risk from underlying mortgages (or other collateral). Because the underlying CDOs were themselves complex structures, a CDO\(^2\) turns out to be even more challenging to value.
CDO market supply and demand
The market for CDOs developed because it offered advantages to both the issuers and the investors. On the issuer side, CDOs provided a way for financial intermediaries to remove assets from their books and thereby reduce their regulatory capital requirements, while continuing to earn fees for originating and servicing the mortgages. Also, some types of CDOs—such as synthetic CDOs—were used for hedging default risks that were on the balance sheets of investment banks for client relations or other purposes.

On the demand side, CDOs offered attractive yields over other like-rated notes. By investing in CDOs, an investor could achieve an overall greater level of asset class diversification while maintaining targeted credit risk levels. The most senior tranches of CDOs were structured to achieve AAA ratings even when the underlying collateral assets were BBB or lower—a result of how the rating agencies viewed collateral protection afforded by diversification and seniority in the capital structure. Some institutional investors could not invest directly in the underlying BBB-rated collateral because of restrictions in their investment guidelines, but they could invest in the highly rated, senior class of notes from a CDO. Alternatively, investors willing to take on more risk could invest in lower tranches issued by the CDO. These subordinate tranches came with lower credit ratings and higher coupon rates.

Synthetic CDOs

The virtual CDO
Whereas a cash-based CDO derives its cash-flows from underlying securities such as ABS, MBS, or other bonds and loans, a synthetic CDO derives cash-flows from CDS (a form of credit insurance) on a basket of reference entities, which could be high-grade corporate bonds, but could also be MBS or ABS. By 2002, US and European synthetic CDOs overtook cash-based CDOs in terms of transaction volume. (Refer to the appendix for a primer on CDS.) During 2006 and 2007, the synthetic CDOs of most relevance to the credit crisis were those effectively insuring various tranches of home equity ABS.

In a synthetic CDO, the issuing special purpose vehicle (SPV) enters into a number of CDS contracts where it sells protection on a reference portfolio. The premiums paid by the counterparties (buyers of protection) in these CDS provide a stream of cash flows to investors in the synthetic CDO. If credit events occur, the contingent payments reduce the cash flow to synthetic CDO investors. If large enough, credit events may result in calls on investors in the “unfunded class” of the synthetic CDO to put up cash to fund the contingent payments. Synthetic CDO payment mechanics, as well as funded and unfunded classes will be described in greater detail below.
One can think of a synthetic CDO as a virtual CDO because, in many senses, it mimics the behavior of an ordinary cash-based CDO that contains the reference portfolio as its collateral. However, in addition to the different source of cash flows, there are two basic features that distinguish synthetic structures from cash-based ones. The first lies in the degree to which the CDO is managed. While there is a range of management styles that one might see in the case of a cash-based CDO, synthetics are typically static. The other main difference lies in the funding of the liabilities. Cash-based CDOs are fully funded, meaning that investors pay in advance for their bonds. The money is used, in turn, to purchase the securities that back the structure. Synthetic CDOs, on the other hand, are only partially funded. In addition to the funded classes, they include an unfunded class (typically the super senior tranches) for which investors need not put any money down initially.

### Funded and unfunded classes

Synthetic CDOs can have funded and unfunded classes. Noteholders of the funded classes purchase interests in the synthetic CDO by making a payment up front. In return, they are entitled to cash flows subject to availability. Assuming no (or limited) credit events, they will continue to receive payments of principal and interest (as derived from premiums from the buyers of protection).

Like cash CDOs, synthetic CDOs have senior-subordinate structuring, as illustrated in Figure 9. Because the funded classes are subordinate to the unfunded ones, they enjoy a higher coupon, but take on a greater risk of losing some or all of their investment. Because of the initial capital payment made by the investor in a funded class, their experience is akin to that of bondholders. By contrast, the investing experience of someone who has an interest in the unfunded class is much more like a seller of protection in a CDS. The unfunded classes form what is called the super-senior tranche of a synthetic CDO. Investors in the super-senior tranche put no money down and receive periodic payments with lower coupons than the funded noteholders. The risk they face lies in their obligation to compensate buyers of protection if sources from the funded class subsequently prove insufficient should severe credit events occur.

**Figure 9. Synthetic CDO structure example**

<table>
<thead>
<tr>
<th>Class</th>
<th>Amount ($MM)</th>
<th>% of Deal</th>
<th>Subordination (%)</th>
<th>Ratings (Moody’s/S&amp;P)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unfunded</td>
<td>240.0</td>
<td>80.0</td>
<td>20.0</td>
<td>Aaa/AAA</td>
</tr>
<tr>
<td>Class A</td>
<td>13.5</td>
<td>4.5</td>
<td>15.5</td>
<td>Aaa/AAA</td>
</tr>
<tr>
<td>Class B</td>
<td>9.0</td>
<td>3.0</td>
<td>12.5</td>
<td>Aa2/AA</td>
</tr>
<tr>
<td>Class C</td>
<td>7.5</td>
<td>2.5</td>
<td>10.0</td>
<td>Baa2/BBB</td>
</tr>
<tr>
<td>Equity</td>
<td>30.0</td>
<td>10</td>
<td>0.0</td>
<td>Not Rated</td>
</tr>
</tbody>
</table>
Payment mechanics
The easiest way to understand the payment mechanics of a synthetic CDO is to start by considering the funded classes. The funded notes are purchased with cash, and in return, the investors receive security from one of the CDO tranches. This instrument is called a credit-linked note (CLN). The CLN, issued by the synthetic CDO trust, is effectively a high-grade bond with an embedded CDS. This means that unless there is a credit event in the reference entities, the investor is entitled to full coupons and the full repayment of his principal. The up-front payments made by investors in the funded classes are then invested in low-risk, highly rated securities, like Treasuries. These securities, now owned by the trust, are set aside as a source of good faith money, akin to margin, for the protection buyers (again, external to the CDO), should there be a credit event. Buyers of the unfunded class receive a security in exchange for a conditional funding commitment. See Figure 10.

Figure 10. Initial cash flows in a synthetic CDO

During the life of the transaction, the cash flow from the insurance premiums is used to meet both the funded and unfunded noteholder’s principal and interest payments. The funded noteholders also receive interest from the highly rated securities purchased by their principal payment. These synthetic CDO cash flows are illustrated in Figure 11a.
What if there is a credit event?
If there is a credit event in the reference portfolio, the CDS contracts require the synthetic CDO to make credit event-contingent payments to buyers of protection. Contingent payments are first funded by interest income from the highly-rated assets, purchased with the initial payments of funded class investors. Consequently, the holders of funded notes receive reduced payments when contingent payments occur. If the credit events are particularly severe, the synthetic CDO might be required to liquidate some or all of the highly rated assets. Note that as long as up-front capital of the funded class is available to meet contingency payments, the unfunded class is immune from losses—consistent with the senior-subordinate structure as described above. Only if the highly rated assets are fully depleted do the unfunded noteholders get called upon to make contingency payments. These post-credit event cash flows are illustrated in Figure 11b.

Figure 11a. Subsequent cash flows in a synthetic CDO assuming no credit events

Figure 11b. Subsequent cash flows in a synthetic CDO assuming credit events
The ABX Index as a Subprime Price Benchmark

A challenge faced when trying to value subprime-backed CDOs is that mortgage-based structured products are traded over the counter (OTC). Information about the prices of various mortgage-backed asset classes usually requires calling dealers to obtain quotes or recent transaction values. In an efficient market, transaction prices reflect publicly available information as processed by the multitude of market participants. Prior to 2006, however, no such exchange with publicly available data existed for a security whose value was dependent on the performance of subprime residential mortgages.

The January 2006 introduction of the ABX.HE indices provided daily, publicly available data. These indices are among a larger family of credit and structured finance indices that are administered by Markit, an organization of dealers in home equity ABS. Each ABX.HE index tracks the value of CDS (which also trade OTC) on specified tranches of home equity ABS. A separate index was initially constructed for each of the following five investment grade ratings: AAA, AA, A, BBB, and BBB-.

The January 2006 indices were followed by three additional versions, or vintages, introduced in July 2006, January 2007, and June 2007. Each index is based on home equity ABS originated in the prior six months. So the January 2006 index, ABX.HE 2006-1 for short, references ABS issued during the last half of 2005. New vintages were then released semi-annually to gather a representative sample of home equity ABS originated since the last index. No new indices were introduced following the decline in the value of the indices after the first half of 2007. Consequently, a total of 20 ABX.HE indices—four vintages and five investment grades within each vintage—were created.

The indices for each vintage are constructed from CDS on tranches of a selected sample of 20 home equity ABS deals. Each ABS deal must have a tranche at each of the five rating levels. Thus, the same 20 deals serve as the reference entities for the CDS that make up the indices for a single vintage. That is, the 2006-1 vintage, released in January 2006, has five indices that are all based on different tranches of the same 20 ABS deals. The index constituents do not change over the life of the index.

These Markit indices are not merely data series that track constituents over time—they also underlie tradable OTC contracts used by broker-dealers and other market participants to hedge, speculate, and trade. (i.e., the indices are used to buy and sell protection against defaults and other credit events.) The protection buyer pays a fixed rate of a set number of basis points, quoted on a per annum basis, to the protection seller each month. The protection seller pays the protection buyer in the case of credit events including interest shortfall, principal shortfall, or a writedown of the underlying home equity ABS.

Importantly, Markit makes daily closing prices available on its website along with some historical data. For each ABX index, Markit solicits closing mid-market transactions prices. Markit then uses an algorithm to compute the daily fixing that is taken from the way the British Banker’s Association calculates LIBOR.
The introduction of the ABX indices added a great deal of visibility to trends in market opinion regarding home equity ABS. By introducing CDS on subprime risk, the ABX created greater liquidity and price transparency than existed for the reference home equity ABS. This is not unique to the ABX. CDS are often so liquidly traded that they respond to market information faster than the bonds they reference. As Longstaff (2008) has stated:

> Despite the lower liquidity of the ABS CDO market, we find that ABX index returns developed significant predictive power for subsequent Treasury bond and stock market returns as the crisis unfolded.33

The availability of market-based prices for credit risk is an important step forward in being able to value CDOs. The 20 ABS deals in any given ABX index will differ from the collateral of any given CDO, and, moreover, the typical CDO will contain many more than 20 ABS deals. As such, there will be basis risk between the movements in the ABX index and a CDO’s collateral securities, even if they are all home equity ABS and of a similar vintage. Although CDO valuation is most often based on models, a model that has as its basic inputs price data from liquid markets will be more reliable. Also, to the extent that the price data from the ABX indices are sensitive indicators of the value of new information coming into the market, they can indicate events that occurred during the market meltdown which had the most impact on perceived credit, counterparty, and liquidity risk.

**The Importance of Correlation and Long Correlation Trade**

To value a CDO, one needs to understand the credit risk of its portfolio of assets. This portfolio credit risk has two parts. First, for each asset, there is the probability that it will default. (Normally, one would also need to make assumptions about the loss severity given default, but we will ignore this aspect to keep matters simple.) Second, there is the default correlation among the assets. For most financial market participants, the first part is easy to grasp (the difficulty is in quantifying it), but the second is more difficult. Before describing how CDOs are valued in more detail, it is worthwhile trying to develop an intuitive understanding of the importance of correlation for the overall credit risk of a diversified portfolio of risky bonds. We will use the analogy of a coin-flipping game to show how correlation can change the risk profile of the tranches in an ABS.

**No correlation of default**

Let us suppose that I-bank develops a very simple CDO of very risky bonds that it divides into two tranches that it sells to two other people—Tranche A Investor and Tranche B Investor. The rules are as follows:

1. I-bank sells shares in the Trust separately to Tranche A (Senior) Investor and Tranche B (Subordinate) Investor.
2. With the money raised, I-bank buys two bonds, Bond 001 and Bond 002, from different issuers. It places the bonds into the Trust as Collateral to be held for Tranche A Investor and Tranche B Investor.
   a. Each bond either matures in one year and pays $100 in principal, or it defaults and pays back nothing.
   b. For each bond, the probability of default is 0.5.

3. At the end of one year, then, the bond principal repayments are distributed to Tranche A and Tranche B according to the following waterfall:
   a. If neither bond defaults then all $200 of the cash from the bond repayments is used to pay Tranche A Investor and Tranche B Investor $100 each (we assume a zero interest rate for simplicity).
   b. If one bond defaults and the other does not, then $100 in cash from the non-defaulting bond principal repayment is paid to Tranche A Investor and Tranche B Investor receives nothing.
   c. If both bonds default then neither Tranche A Investor nor Tranche B Investor receive anything.

Admitting these are very risky investments because of the high default rates, it is worthwhile trying to figure out the value of each tranche; the two investors might not actually want to pay a full $100. If both investors are risk neutral, then how much should Tranche A Investor be willing to pay for Tranche A? And how much should Tranche B Investor be willing to pay for Tranche B?

To analyze this further, let us put the probabilities of repayment versus default into a simple cross-tabulation.

<table>
<thead>
<tr>
<th>Bond 001</th>
<th>Bond 002</th>
</tr>
</thead>
<tbody>
<tr>
<td>Repay</td>
<td>Default</td>
</tr>
<tr>
<td>Repay</td>
<td>?</td>
</tr>
<tr>
<td>Default</td>
<td>?</td>
</tr>
<tr>
<td></td>
<td>0.50</td>
</tr>
</tbody>
</table>

Recall we are assuming there is 50-50 chance of default for each of the bonds. But knowing this does not by itself tell us the probability of the outcomes of interest. What we need to know is the joint probability of (Repay, Repay), (Repay, Default), (Default, Repay), and (Default, Default). In other words we need to fill in the cells inside this matrix where we now have question marks.

To fill in the cells, we need more information. Based on what we know, the only requirements for these joint probabilities are that: the sum of the joint probabilities in each row (that is the sum of the two cells in each row) equals 0.5; the sum of the joint probabilities in each column (that is the sum of the two cells in each column) equals 0.5; and all joint probabilities are non-negative.
If we were to think of the bond payoffs as being two separate coin flips, we would then assume that the default rates of Bond 001 and Bond 002 are independent of each other—in other words, that they are uncorrelated. A property of independent probabilities is that the probability of a joint outcome (such as Default-Default) is the product of the probability of each of two separate outcomes (the probability of Bond 001 defaulting times the probability of Bond 002’s defaulting is like the probability of flipping two Tails). Consequently, we can fill in the table as follows:

<table>
<thead>
<tr>
<th></th>
<th>Bond 002</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Repay</td>
<td>Default</td>
</tr>
<tr>
<td>Bond 001</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Repay</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>Default</td>
<td>0.25</td>
<td>0.50</td>
</tr>
<tr>
<td></td>
<td>0.50</td>
<td>1.00</td>
</tr>
</tbody>
</table>

We are now in a position to determine the probabilities of each of the following outcomes:

1. Neither Tranche A nor Tranche B gets wiped out  
   \[ P(\text{Repay, Repay}) = 0.25 \]

2. Tranche A gets wiped out  
   \[ P(\text{Default, Default}) = 0.25 \]

3. Tranche B gets wiped out  
   \[ P(\text{Default, Default}) + P(\text{Repay, Default}) + P(\text{Default, Repay}) = 0.75 \]

Consequently, the expected payoffs to Tranche A and B are as follows:

- Tranche A expected payoff  
  \[ (1 - .25) \times 100 = 75 \]

- Tranche B expected payoff  
  \[ (1 - .75) \times 100 = 25 \]

Therefore, Tranche B Investor should pay no more than $25 for Tranche B and Tranche A Investor should pay no more than $75 for Tranche A.

One would be skeptical, however, of the assumption of no correlation of default—bond payouts are not determined by coin flips, but by economic factors. For example, macroeconomic circumstances might be expected to affect the prospects of both Bond 001 and Bond 002; in a declining economy, both would be expected to become less creditworthy than in a growing economy. Such skepticism is warranted because if there is high default correlation, either positive or negative, then the effects on the expected payoffs to tranche holders can be quite profound.
Positive default correlation

One can think of various economic processes that could cause correlation. For example, consider a set of economy-wide factors, other than the idiosyncratic management of the issuers of the bonds, which affect the probability of default. Under condition growth both Bond 001 and Bond 002 are able to repay their entire principal, but under recession both bonds default and repay none of their principal; if condition growth occurred 50% of the time and condition recession occurred 50% of the time, then for each bond we would observe that the chances of repayment or default are still 50-50; but the outcomes are now positively correlated.36

We assume that when the shares in the tranches in the CDO are sold to investors, it is equally likely that there will be either growth or recession; but which condition will occur is not known in advance. The default probabilities then have the highest level of positive correlation. The following table fills in the joint probabilities for the various outcomes for Bond 002 and Bond 001. Note that the probability of default for each bond individually is the same as in the case when the default rates were independent events (zero correlation).

<table>
<thead>
<tr>
<th></th>
<th>Bond 002</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Repay</td>
<td>Default</td>
</tr>
<tr>
<td>Bond 001</td>
<td>0.50</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>0.00</td>
<td>0.50</td>
</tr>
<tr>
<td></td>
<td>0.50</td>
<td>0.50</td>
</tr>
<tr>
<td></td>
<td>1.00</td>
<td></td>
</tr>
</tbody>
</table>

We are again in a position to determine the probabilities of each of the following outcomes:

1. Neither Tranche A nor Tranche B gets wiped out
   \[ P(\text{Repay, Repay}) = 0.50 \]

2. Tranche A gets wiped out
   \[ P(\text{Default, Default}) = 0.50 \]

3. Tranche B gets wiped out
   \[ P(\text{Default, Default}) + P(\text{Repay, Default}) + P(\text{Default, Repay}) = 0.50 \]

Consequently, the potential payoffs to Tranche A and B are as follows:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Tranche A payoff</td>
<td>$(1 - 0.50) \times 100 = 50$</td>
</tr>
<tr>
<td>Tranche B payoff</td>
<td>$(1 - 0.50) \times 100 = 50$</td>
</tr>
</tbody>
</table>

Tranche B Investor should pay no more than $50 for Tranche B and Tranche A Investor should pay no more than $50 for Tranche A. The shares in Tranche A are worth less than they were when there was no correlation ($50 versus $75), while the shares for Tranche B are worth more ($50 versus $25). As a general matter, increasing correlation will reduce the value of senior tranches while increasing the value of subordinate tranches, and vice versa, all else equal.
Now let us suppose that the senior tranche holder, Tranche A Investor, had originally paid $75 for Tranche A shares under the assumption that the probabilities of default were independent (that is, believing that the CDO was diversified and relatively less risky than owning a single bond). If this assumption was mistaken, and the default outcomes were positively correlated, then the value of the investment would be less than $75. In this simple example, there is still a 50% chance that Tranche A Investor will earn back his principal and make $25 profit from the investment. But if the Tranche A investor invested in many of these securities, all under the assumption that the underlying bonds were diversified and their defaults uncorrelated, he would very likely lose money.

The long correlation trade
So far, we have seen how investors can be long a tranche in a cash or synthetic CDO by investing in the CDO, or they can be short a tranche in the CDO by being a buyer of protection on the cash CDO or a buyer of protection on a synthetic CDO (this concept is discussed later in the appendix). An investor can also undertake positions with a view that default correlations are higher than is reflected in the market price. The investor can do this by buying the equity tranche and shorting the higher rated tranche.

To see how this can be done, consider the example above where the market believes the underlying bonds are uncorrelated and has priced Tranche A and B accordingly. Suppose an investor believes strongly that the two bonds are perfectly correlated, contrary to the market’s assumption. Acting on this belief, the investor can make a long correlation trade. To do this, the long correlation investor first buys Tranche B, effectively the equity tranche in this example, for $25. Other investors buy Tranche A for a total of $75. The long correlation investor then acquires a CDS on Tranche A, equal to $100 in notional value, which requires an initial premium payment of $25.37

Consider now the only two possible outcomes under perfect correlation: both bonds pay $100 or both pay $0:

1) If both bonds pay $100 then both Tranche A investors and the Tranche B investor receive $100. Thus the long correlation trade nets $50 (= $100 received on the Tranche B payment—$25 paid in Tranche B principal—$25 premium on the CDS for Tranche A).

2) If both bonds default, then both Tranche A investors and the Tranche B investor receive $0 on their CDO investments. But the CDS pays $100, yielding a net gain of $50 to the long correlation trader (= $100 received on the CDS contingent payment—$25 paid in Tranche B principal—$25 premium on the CDS for Tranche A).

Consequently, if the actual correlation is greater than expected, the long correlation trade nets $50. If the long correlation trader knows the true correlation, then the trade is a risk-free arbitrage.
If, alternatively, the defaults were actually uncorrelated, as believed by the market, then the long correlation trade has a 50% chance of netting $50 and a 50% chance of losing $50—that is, it is a fair game. The two outcomes above, each netting $50, will each occur with only a 25% chance. There are two other outcomes that can occur with a 25% chance each and in both of these outcomes Tranche A is repaid $100 and Tranche B is paid $0; meaning the long correlation trade loses the $25 paid for Tranche B as well as the $25 premium for the CDS on Tranche A.

Valuation of Collateralized Debt Obligations with Subprime Collateral

With some intuition for the importance of correlation, we now can describe the process for valuing actual CDOs, in which correlation is a critical issue, though only one of several. The probability of default of a given asset and correlation of defaults across assets are the key factors that determine the overall credit risk for the collateral pool underlying a CDO. Another source of risk for CDOs backed by mortgages, as described earlier, is prepayment risk. The cash flows from the collateral, which would otherwise be fixed payments, must be adjusted for these sources of risk. The resulting payments are then passed through the waterfall structure, the other component of the modeling process. We focus first on modeling the riskiness of the cash flows.

The borrowers’ right to prepay means that there is uncertainty about when investors in a CDO will be repaid their principal. As described above, subprime collateral usually comes in the form of hybrid loans with relatively short fixed-rate periods (two to three years). Given subprime borrowers’ increased likelihood of default, the risk to the investor is that principal will not be prepaid upon reset, and that the risky underlying collateral will remain outstanding for extended periods. Because they are typically issued with a floating rate coupon, the sensitivity of CDO prices to interest rate moves (the “effective duration”) is minimal.

Methods to model or forecast prepayments come in all forms of complexity, and are usually done at the level of the individual mortgage loans. Whatever the modeling approach, interest rates and housing price appreciation are major drivers of borrower’s ability to refinance and prepay. As discussed above, prepayment and default risk are closely linked for subprime mortgages, since subprime borrowers who cannot refinance because of declines in the values of their homes will face ballooning mortgage rates (in many cases, when two-year teaser rates expire) and be more likely to default. The result will be subsequent defaults in the associated ABS, and the CDOs that hold those securities.

Default risk is due to uncertainty about whether a particular CDO debt class will repay full principal and interest. As alluded to above, there are two major components in estimating default risk for a CDO debt class: the default probabilities for each of the underlying securities of the CDO and correlations between defaults of those securities. The default probability tells us the chance of an individual security in the CDO defaulting in a given amount of time. The default correlation tells us—as in the simple example in the previous section—about the joint probabilities; the chances of multiple securities defaulting at the same time.
Estimating default probabilities

The key to estimating default probabilities for the underlying assets of a CDO is the nature of the data available. There are typically two possible sources: historical data and current market data. If the collateral consists of subprime mortgages (or more precisely, tranches of securities backed by subprime mortgages), historical data might be in the form of past loan performance data, perhaps averaged in a weighted fashion across a pool of mortgages. An analyst could use this past history of loan performance to estimate the historical rate of default for the collateral in his deal. The crucial step would then be to assume that the future will be similar to the past, so as to be able to apply the estimate to the valuation at hand. In other words, the downside to historical estimates is that they are slow to capture sudden changes in the economy.

Market-based data offer an alternative approach to estimating default probabilities that avoids some of these difficulties. Because CDO collateral is rarely actively traded with publicly reported prices, market data would usually consist of credit spreads from some widely followed index, such as the ABX if the underlying collateral comprises subprime mortgages, with the rating, vintage, and so forth of the index being matched as closely as possible to the collateral. This assumes that the components of the index are representative of the actual collateral backing the deal. For example, the ABX indices are often considered to be a good representation of subprime deals of similar vintages and ratings, though there are some known limitations of the indices’ coverage which should be taken into account.

Armed with spread data from an appropriate index, there are established methods for converting spreads into default probabilities. These estimates would have the advantage over the historical values of being forward looking. Given the fast-moving nature of market events during the credit crisis, this is of critical importance. As noted above, indices such as the ABX have been at the center of many of the events of the current crisis, and being able to calibrate the model to these indices is a useful way to reflect current market sentiment in the valuation. An additional benefit is that using current market data to estimate the inputs is more technically sound and defensible, grounded in the standard, widely-accepted theory for the valuation of derivatives.

Estimating default correlations

Although default correlations across portfolio assets are critical to a CDO’s valuation, the estimation of default correlations has long been a difficult problem, stemming from the fact that a default is a rare event. Thus historical data on defaults do not provide a reliable way to estimate correlations. This leaves market data as the only available avenue. However, true market data on correlations—that is, data that are current and forward looking—are generally not available, either. In practice, what is generally done is to use historical co-movements of publicly traded assets over some period of time to estimate correlations. For instance, for corporate debt, an estimate of default correlations can be derived from historical correlations of equity returns. To estimate correlations across different types of home equity ABS (e.g., home equity ABS of different ratings), the historical correlations of the spreads of different series of the ABX may be used. It should be noted that such estimates are essentially historical in nature, depending as they do on past data over some period of time, even though the data come from publicly traded asset prices.
Simulating the cash flows

With the necessary inputs in hand, one can simulate randomly occurring defaults in the collateral and hence the cash flow streams that are paid to investors in a particular tranche of the CDO. This method is called Monte Carlo simulation. Each scenario is a random draw from a given probability distribution of future default experience for the bonds making up the CDO’s collateral. In each scenario, the simulated defaults determine the principal and interest proceeds paid from the collateral pool, and these proceeds are sent through the CDO’s cash flow waterfall. Typically, thousands of future cash flow scenarios may be generated, and then discounted back to the present to create an expected value for each scenario. These many scenarios are then averaged to create an expected discounted cash-flow (DCF).

Figure 12. Two scenarios of simulated defaults

Figure 12 shows two simulated scenarios of defaults of the underlying bonds of a simple CDO with 10 underlying ABS bonds and 10 periods to maturity. In the first scenario (labeled “Few Defaults”), the computer simulation generated only two defaults over the life of the CDO. In the second scenario (“Many Defaults”), seven underlying bonds default before the CDO matures. Each of these scenarios generates a stream of principal and interest payments to the tranches, as shown in Figure 13. Panel (a) shows how the outstanding balance of the collateral pool and the principal due to the notes declines over time in the Few Defaults scenario. If there were no defaults at all, the height of the collateral balance bar would still decline as principal was paid from the collateral to the CDO, but it would always remain at least as high as the total tranche balance. The effect of the defaults is to reduce the collateral balance below the total tranche balance (which first occurs with the default in period 3). Because there are few defaults in this scenario, at the end of the CDO’s life, all of the rated tranches have been repaid. Panel (b) shows that, in the Many Defaults scenario, there was so much collateral loss that only the A tranche was paid back entirely, while the B, C, and equity tranches had principal left unpaid.
Figure 13a. Simulated payments for two cash flow scenarios
Collateral and Tranche Balances (Few Defaults)

Figure 13b. Simulated payments for two cash flow scenarios
Collateral and Tranche Balances (Many Defaults)
Figure 13c. Simulated payments for two cash flow scenarios
Total interest paid and due (Few Defaults)

Figure 13d. Simulated payments for two cash flow scenarios
Total interest paid and due (Many Defaults)
Panels (c) and (d) compare the interest payments due to the tranches with the total interest paid. The CDO is normally set up so that the interest paid by the collateral bonds is greater than the coupons on the tranches. The difference is called “excess interest,” and is often paid to the equity tranche after the other tranches have been paid; there is no interest or principal “due” to the equity tranche. Note that even if there is no overcollateralization explicit in the deal structure at inception, excess interest can serve to create it. By using the excess interest to pay down principal of senior tranches, it is possible to build up a pre-specified level of overcollateralization.

When collateral defaults, there are fewer bonds paying interest to the CDO and the interest coming into the CDO may be less than the interest due to the noteholders. Panel (c) shows that, in the Few Defaults scenario, there is always enough interest to pay the noteholders their coupons, even though there are some defaults. In the Many Defaults scenario, though, shown in panel (d), the large number of defaults leads to interest shortfalls, starting in period 6, when the C tranche does not receive its due interest, and continues until, in periods 9 and 10, both the B and C tranche do not receive their due interest.

We would then compute the discounted present value of the principal and interest payments in each of these scenarios (with the value in the Many Defaults scenario being lower than that of the Few Defaults scenario). Repeating this process many times over and averaging the results provides the expected value of each of the CDO tranches.

**Modeling the waterfall structure**

Each simulation provides a future cash flow scenario—a stream of future cash flows paid to the CDO from the underlying securities, with these cash flows determined by defaults and prepayments simulated in that scenario. On each date that the CDO pays its investors, these cash flows are sent through the CDO’s cash flow waterfall. The “waterfall” is the term used to describe how payments from a CDO’s collateral are distributed to investors. The details of the waterfall are described in the CDO’s prospectus and often can be quite detailed, as the distribution of proceeds can depend on many different contingencies.

For example, a simple cash flow waterfall for a CDO would incorporate: (1) fees paid to collateral managers, and other administrative expenses, (2) payments of interest and principal to the noteholders, from the most senior to the most junior tranches, and (3) diversions of cash flows from junior to senior tranches after the failure of coverage tests.

The coverage tests of part (3) are tests performed to determine whether the CDO collateral is sufficient to make the required future principal and interest payments to the noteholders. For example, a senior over-collateralization test would test whether there is sufficient collateral balance remaining such that the senior tranches could still expect repayment of their principal. Usually when a test like this fails, the portion of interest received from the collateral that would normally go to the junior tranches is instead used to pay down the senior tranche principal until it is low enough that the test is passed. A CDO might have a similar coverage test that applies to the interest paid by the collateral. It might also have several variants of principal and interest coverage tests, with different rules for determining the distribution of cash flows when they fail. A careful review of a CDO’s prospectus is required to model these tests accurately.
While most CDOs have a waterfall similar to the simple one described above, each CDO also has many of its own particular terms and details that affect the distribution of payments. Although simple waterfall models are often a good first approximation for valuing a CDO, more accurate valuations require taking into account all of the particulars of the CDO at issue.

Lastly, issuers, trustees, and collateral managers are often given discretion in their roles that can affect the valuation of a CDO. For example, collateral managers usually have at least some discretion in buying and selling securities, so a CDO’s collateral will change over time in ways that cannot be easily modeled.

Conclusion

After lower-rated subprime securities began to decline in value in the spring of 2007, the market for structured products broke down more broadly in August 2007. Because a significant portion of CDOs were constructed from home equity ABS, the credit crisis of 2007 had a severe impact on the performance of these structured products. Some market value losses were credit-related, but initially much was related to decreased liquidity from institutional deleveraging. As such, CDOs are now central to a great deal of securities litigation, and most likely will continue to be so for some time to come. As discussed here, the shortage of market data means that valuation will often require modeling CDO cash flows. Due to the structure of CDOs, however, this can be a complex endeavor. In this paper we have provided a guide to the economics and structure relevant to the valuation of structured finance CDOs for those interested in some of the intricacies of CDO-related litigation.
Appendix
Credit Default Swaps

Credit default swaps (CDS) were first introduced in the mid-1990s as a mechanism for institutions to hedge credit risk. CDS on ABS have been around since the late 1990s; however, these contracts were not standardized until 2005, when the dealer template for transacting ABS CDS was first published. In this appendix, we explain the background, mechanics, and pricing behind these CDS. The ABX index, which began trading in January 2006 and is discussed above, tracks the value of CDS on a set of 20 underlying home equity ABS tranches. For the CDS comprising the ABX, writedowns are treated as Floating Amount Events. CDS on corporate names are similar to their MBS counterparts in both structure and economics, but have simpler contractual terms, so we begin our discussion with them.

Insuring what you don’t own
In many ways, a CDS is like an insurance policy. Formally, it is a contract in which a CDS protection seller agrees to make a payment to a CDS protection buyer if some sort of credit event (such as bankruptcy) occurs. In order to minimize potential disputes between the CDS parties, the International Swaps and Derivatives Association (ISDA) has defined a set of six items that constitute a credit event for a corporate name. These are bankruptcy, failure to pay, obligation default, obligation acceleration, repudiation and moratorium, and restructuring. While CDS were initially customized arrangements between individual parties, ISDA helped increase liquidity in the CDS market by creating a standard contractual framework. The ISDA Master Agreement defined credit events and payment procedures, but can be customized by counterparties structuring a particular CDS transaction. Today, to enter into a CDS, one would typically contact a broker who would set up the deal.

As has become a prominent issue since the failure of Lehman Brothers, CDS contracts also involve counterparty credit risk. This is the risk that the counterparty in a transaction will not be able to perform their contractual obligation to pay a premium or make a contingent payment. Counterparty credit risk also factors into CDS valuation, and there are methods for mitigating counterparty risk. However, to keep things simple, this section will not address counterparty risk.

Unlike a typical insurance policy, a CDS allows one to buy protection against credit events concerning reference entities that one does not own. This can lead to a situation where the total outstanding notional amount of CDS trading on a company’s debt can exceed the amount of the actual debt. That is, it is possible for there to be more debt “insured” than actual debt to insure.
**CDS mechanics**

CDS contracts have maturities that range between one and 10 years, with five-year maturity CDS being the most liquid. Over this period, the buyer of protection pays a periodic premium to the seller of protection. If a credit event occurs, then the CDS seller must compensate the CDS buyer by an amount whose size depends on the severity of the event. Depending on the situation, the CDS buyer might either choose physical delivery or cash delivery, although physical delivery is more common because it is often difficult to obtain quotes for the distressed reference credit. In the case of the former, the seller must purchase a bond (issued by the reference entity) at par value. Cash settlement involves the seller paying the buyer a sum determined by the decline of the reference entity’s debt securities.

Consider the example illustrated in Figure 14. The Acme Bank holds bonds of the Luxe motor company. In order to hedge against possible credit events that would lead to reduction in the market value of their bonds, Acme Bank decides to purchase protection in the form of CDS. It goes to a broker who then seeks a counterparty willing to sell this protection. (In many cases, the broker itself serves as the counterparty.) The Veritas Hedge Fund is bullish on Luxe and feels that the likelihood of a credit event is low. After it agrees to sell protection, Veritas immediately starts to receive premiums from Acme. Over the maturity of the swap, Veritas’s gamble will pay off if Luxe (which is external to the CDS) does not experience a credit event: Veritas will receive premiums from Acme but make no contingent payment. If, however, Luxe experiences a credit event, Veritas must compensate Acme.

**Advantages to the CDS buyer and seller**

From the perspective of the seller of protection, one is only obliged to pay future credit event losses, should they occur. In other words, while the seller of protection may need to post collateral, there is no up-front purchase required to enjoy monthly payments of premium. Even though there is a stream of payments that resemble fixed-income coupons, the investing experience is distinctly different from that of a bondholder. For the buyer of protection, there is the advantage of being able to go short credit without the difficulties of shorting a bond.

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**Figure 14. Example of a credit default swap**

<table>
<thead>
<tr>
<th>Acme Bank (Protection Buyer)</th>
<th>CDS Premiums</th>
<th>Veritas Hedge Fund (Protection Seller)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Contingent Payment</td>
</tr>
<tr>
<td>Luxe Motor Company</td>
<td></td>
<td>Credit Event</td>
</tr>
</tbody>
</table>
Investing in a CDS allows one to take on a pure credit position. By this, we mean that the value of a CDS portfolio will be directly affected only by changes in the creditworthiness of the reference entity. In contrast, the value of a portfolio of bonds, in addition to potentially having credit risk, will also be directly affected by interest rate moves. Of course, interest rates do influence credit spreads, so CDS are not immune from interest rate moves, but they are only indirectly affected.

As the MBS and ABS markets grew, it was realized that the procedures and definition used for corporate CDS were inadequate for these securities. In 2005, ISDA developed a new agreement template that better fit the characteristics of these securities. The new template was called the “Pay-as-you-go (PAUG) or physical settlement” form. Unlike in the case of corporate bonds, where a single CDS covers a whole set of the company’s bonds, an MBS CDS usually only references a single tranche of a single MBS.

Under the terms of PAUG, a CDS isn’t settled and terminated once a credit event occurs. Instead, there are a few types of “Floating Amount Events,” where the protection seller pays the protection buyer the value of the event, and the contract continues until the reference obligation itself terminates. The Floating Amount Events are (1) Interest Shortfall, where the MBS tranche fails to pay all due interest to the noteholders in a period; (2) Principal Shortfall; and (3) Writedown, where the issuer of the MBS reduces the principal amount to be paid back to the note holder because of losses in the collateral. In these cases, the protection seller would pay the protection buyer either the amount of the shortfall or writedown each time such event occurred.

Figure 15. **Example of CDS on a home equity ABS bond**

Figure 15 shows an example of how the CDS would work. Notice that there is an extra payment made from the protection buyer to the seller, labeled “reimbursement.” If an interest shortfall in one period is eventually repaid to the note holder in a later period, then the protection buyer must reimburse the seller the floating amount payment paid when the interest shortfall occurred.
There are events that can cause the CDS to be physically settled and terminate. As in the corporate bond case, these are called credit events. Specifically, the events are: (1) Failure to Pay Principal (usually either at maturity, or when the security liquidates); (2) Writedown; and (3) Distressed Ratings Downgrade, where the reference bond is downgraded to Caa2 or below. Note that writedowns can be settled either by Floating Amount payments or physical settlement.

At the time that a CDS is arranged, the premium is set to be equal to the present value of the expected contingent payments. In theory, the value of the CDS to either party should be exactly zero at the time of origination. The ABX reflects this by starting at 100 at initiation. The estimation of contingent payments requires that one make assumptions of default rates and recovery rates. However, these quantities are heavily dependent on market conditions, so one expects them to change over time. As they change, either the buyer or the seller of protection will have a security that may be worth more or less than its original value of zero. If a CDS position is positive at some point in time before maturity, it can be sold for a profit. If, however, a CDS position has a negative value, one must pay to exit the position. The ABX price reflects this value by falling below 100 as the value of the protection seller’s leg decreases (and the value of buyer’s leg increases) from its initial zero value. As the value of the seller’s leg increases above its initial zero value, the index price rises above 100. The difference between 100 and the ABX price represents the up-front payment that must be made to a protection seller to buy protection on the reference bonds at the fixed premium set at the index’s initiation. If the index is priced over 100, the seller must make an up-front payment to a protection buyer.
Notes

1 The opinions expressed herein do not necessarily represent the views of NERA Economic Consulting or any other NERA consultant. Please do not cite without explicit permission from the authors.

2 This paper expresses the author’s views and does not necessarily reflect those of the Commission, the Commissioners, or other members of the staff.

3 NERA proprietary database.


6 See, for example, Alan Greenspan’s remarks to the Federal Reserve Bank of Chicago in May 2005; a speech titled “Risk Transfer and Financial Stability,” http://www.gov/federalreserve/boarddocs/speeches/2005/20050505/default.htm:

“Two years ago at this conference I argued that the growing array of derivatives and the related application of more-sophisticated methods for measuring and managing risks had been key factors underlying the remarkable resilience of the banking system, which had recently shrugged off severe shocks to the economy and the financial system. […] As is generally acknowledged, the development of credit derivatives has contributed to the stability of the banking system by allowing banks, especially the largest, systemically important banks, to measure and manage their credit risks more effectively.”


9 There is a veritable alphabet soup of terms used to refer to securitizations of mortgages, including CMO (Collateralized Mortgage Obligation), and REMIC (Real Estate Mortgage Investment Conduit). Some of these terms refer to the trust or special purpose vehicles that issue the bonds and not the bonds themselves. One important example, though, are bonds backed by subprime mortgages, and labeled as ABS (Asset-Backed Securities).


13 For an overview of these credit enhancement options, see The Handbook of Mortgage-Backed Securities (Frank J. Fabozzi, ed., 6th edition, 2006).

14 An exception could arise if there were an unforeseen shortfall in cash flows, as payments made to subordinated bondholders cannot be clawed back. Consider the following simplified example of an ABS with two payment periods (with expected equal payments) and two classes, a senior class with 20% subordination and a junior class. In the first payment period, the ABS received 100% of cash flows due and anticipated no shortfall in the second period, so paid 80% of cash flows to the senior class and 20% to the junior class. In the second period, cash flows unexpectedly fall to 75%, all of which are paid to the senior class. Although 88% of total cash flows are received, the senior class experiences a shortfall of 2.5%.

15 Depending on when the losses become anticipated, the junior or Class C tranche may have already received some principal payments, such that the principal available to Class B bondholders would be reduced by that amount.


17 In July 2007, Morgan Stanley estimated that subprime mortgages represented 46% of the collateral in high grade structured finance CDOs, with potential additional exposure via the 20% of collateral in tranches of other CDOs, and 77% of the collateral in mezzanine structured finance CDOs. “Ratings Action: Something Had to Give,” Morgan Stanley CDO Market Insights, July 16, 2007.

18 The underlying MBS are already assembled with an eye towards diversification in its mortgage pool: by geography, by services, etc. By investing in MBS issued by different mortgage originators, though, CDOs can diversify their mortgage exposure even more.

19 Gary Gorton, “The Panic of 2007,” Fed. Res. Bank of Kansas City Jackson Hole Conference (August 2008, August 4, 2008 ver.), available at http://www.kc.frb.org/publicat/sympos/2008/gorton.08.04.08.pdf at 3. (“The key security design feature of subprime mortgages was the ability of borrowers to finance and refinance their homes based on the capital gains due to house price appreciation over short horizons and then turning this into collateral for a new mortgage (or extracting the equity for consumption). The unique design of subprime mortgages resulted in unique structures for their securitization, reflecting the underlying mortgage design. Further, the subprime residential mortgage-backed securities (MBS) bonds resulting from the securitization often populated the underlying portfolios of collateralized debt obligations (CDOs).”)...

20 Authors’ calculations based on four bonds from each of the ABX-06-1 and 07-1 indices, where bonds are all those for whom the necessary data was publicly available.

21 Or, if the homeowner then qualified, a conforming loan.

22 Many CDOs are over-collateralized as a form of credit enhancement. For example, if $100 were owed to bondholders, the CDO would have $105 in collateral and any cash flows in excess of $100 would flow to the equity tranche holders. In CDOs without overcollateralization, equity tranche holders would receive payment only if $100 in collateral yielded more than $100 in cash flows.
All equal risk diversification is a way to achieve a higher rating. See U.S. CMBS: Sensitivity of CRE CDOs to Key Credit Variables (Moody’s Investor Service), Sept. 21, 2006.

Assuming that the CDOs held by a mezzanine CDO were also holding the typical amount of RMBS, this implies that the typical mezzanine CDO held about 94% subprime or other RMBS.

Basel Committee on Banking Supervision, Credit Risk Transfer: Developments from 2005 to 2007 (April 2008).


This portfolio of high-grade securities is called, somewhat confusingly, collateral. In the cash CDO, the collateral is, say, the actual tranches of the MBS owned by the CDO. In the synthetic CDO, these same tranches of MBS would be called the reference portfolio.

Corporate and municipal bonds also trade over the counter. However, all transactions must be reported. Corporate bond trades must be reported to the TRACE system within 15 minutes and is reported to other dealers in real time; municipal bond trades must be reported to the MSRB Trade Reporting System by midnight of the same day. All reported prices are publicly available within a day.

The other major credit indices are described in a following section. The initial dealers were: Bank of America, Barclays Capital, Bear Stearns, BNP Paribas, Citigroup, Credit Suisse, Deutsche Bank, Goldman Sachs, JP Morgan, Lehman Brothers, RBS Greenwich Capital, Merrill Lynch, Morgan Stanley, UBS, and Wachovia.


Investors are “risk neutral” when they only care about the expected return of an investment. They always prefer a portfolio with higher expected returns, regardless of how risky the portfolio is, and they are indifferent between two portfolios with the same expected return. A “risk neutral” investor, on the other hand may prefer a portfolio with a lower expected return, if it is also less risky.

As a check, it is worth noting that the sum of these payments is $100. That is, the issuer of the CDO would have a total of $100 to purchase the two bonds. Because each bond has a 50-50 chance of paying either zero or $100, the expected value of each bond is $50 which, therefore, would also be the market value for each. Consequently, the amount raised from the investors just covers the market value of the bonds in the CDO.

In fact they are now perfectly correlated. There’s a 100% probability that either both repay or both default and zero probability of one having a different outcome than the other.

Because the market expects a 25% chance of a default, the premium necessary to receive a $100 payment is $25. In practice, the long correlation trader would use the coupon payments from the equity tranche to fund premium payments on the senior tranche.

It is necessary to use an index because for assets such as home equity ABS there are typically no loan or security-specific market indicators available that are actively traded.


See, e.g., John Hull, Options, Futures and Other Derivatives 505 (7th ed. 2009); Christian Bluhm, Ludger Overbeck and Christoph Wagner, An Introduction to Credit Risk Modeling 205 (2003).

One exception is for corporate debt, where market participants do routinely consider the average “implied correlation” of the CDX index. See Pierre Collin-Dufresne, “A Short Introduction to Correlation Markets,” 7(1) Journal of Financial Econometrics 12 (2009). This is not applicable to home equity ABS, however.

Hull and White, supra, note 40, at 15.

“Credit Default Swaps: hedging towards a more stable system;” Deutsche Bank Research; December 21, 2009.


“Synthetic ABS 101: PAUG and ABX.HE,” Nomura Fixed Income Research, March 7, 2006. (The document bears the date March 7, 2005, but that is clearly incorrect because the ABX index was only launched in January 2006.)


Credit Default Swap (CDS) Primer (Nomura Fixed Income), May 12, 2004.

Credit Default Swap (CDS) Primer (Nomura Fixed Income), May 12, 2004.

ISDA also introduced a cash/physical settlement form, which was intended primarily for use in Europe. See Edward J. O’Connell & Emily H. Goodman, “ABCs of Synthetic ABS,” Journal of Structured Finance (Spring 2006).


Id.
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Contacts
For further information and questions, please contact the author:

Dr. Thomas Schopflocher
Senior Consultant
+1 212 345 1998
thomas.schopflocher@nera.com